MATEMÁTICAS II
ÁLGEBRA **JULIO 2016 B**
PROBLEMA 28

Problema B.1. Se dan las matrices $A=\begin{pmatrix}1&1&-1\\1&2&1\\0&1&1\end{pmatrix}$, $B=\begin{pmatrix}0&1&-1\\2&1&2\\1&0&-1\end{pmatrix}$ e $I=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$.

Obtener **razonadamente**, **escribiendo todos los pasos del razonamiento utilizado**:

a) El determinante de las matrices $A\cdot(2(B)^2)$ (1,5 *puntos*)
y $A\cdot(2(B)^2)\cdot(3A)^{-1}$. (1,5 *puntos*)

b) Las matrices A^{-1} (2 *puntos*)
y $((B\cdot A)^{-1}\cdot B)^{-1}$. (2 *puntos*)

c) La solución de la ecuación matricial $A\cdot X+B\cdot X=3I$. (3 *puntos*)

a) $|A(2(B)^2)| = |A|\cdot|2B^2| = |A|\cdot 2^3|B^2| = 8|A|\cdot|B|^2 = 8\cdot(-1)\cdot 5^2 = -200$

$|A\cdot(2(B)^2)(3A)^{-1}| = |A|\cdot|2(B)^2|\cdot|(3A)^{-1}| =$

$= |A|\,2^3|B|^2\cdot\dfrac{1}{|3A|} = 8\cancel{|A|}\cdot|B|^2\cdot\dfrac{1}{3^3\cancel{|A|}} = \dfrac{8}{27}|B|^2 = \dfrac{8}{27}\cdot 5^2 = \dfrac{200}{27}$

$$\left[\;|A| = \begin{vmatrix}1&1&-1\\1&2&1\\0&1&1\end{vmatrix} = 2-1-(1+1) = -1 \qquad |B| = \begin{vmatrix}0&1&-1\\2&1&2\\1&0&-1\end{vmatrix} = 2-(-1-2) = 5\;\right]$$

b) $A^{-1} \to |A| = -1$

$$Adj(A) = \begin{pmatrix} \begin{vmatrix}2&1\\1&1\end{vmatrix} & -\begin{vmatrix}1&1\\0&1\end{vmatrix} & \begin{vmatrix}1&2\\0&1\end{vmatrix} \\ -\begin{vmatrix}1&-1\\1&1\end{vmatrix} & \begin{vmatrix}1&-1\\0&1\end{vmatrix} & -\begin{vmatrix}1&1\\0&1\end{vmatrix} \\ \begin{vmatrix}1&-1\\2&1\end{vmatrix} & -\begin{vmatrix}1&-1\\1&1\end{vmatrix} & \begin{vmatrix}1&1\\1&2\end{vmatrix}\end{pmatrix} = \begin{pmatrix}1&-1&1\\-2&1&-1\\3&-2&1\end{pmatrix} \to (Adj(A))^t = \begin{pmatrix}1&-2&3\\-1&1&-2\\1&-1&1\end{pmatrix}$$

$$A^{-1} = \begin{pmatrix}-1&2&-3\\1&-1&2\\-1&1&-1\end{pmatrix}$$

$((B\cdot A)^{-1}\cdot B)^{-1} = B^{-1}\cdot((BA)^{-1})^{-1} = B^{-1}\cdot(B\cdot A) = (B^{-1}\cdot B)A = A$

Prop. inversas: $(A\cdot B)^{-1} = B^{-1}\cdot A^{-1}$

c) $AX + BX = 3I$

$$(A+B)X = 3I \rightarrow (A+B)^{-1}(A+B)X = (A+B)^{-1} \cdot 3I$$

$$X = (A+B)^{-1} \cdot 3I = 3(A+B)^{-1}$$

$$A+B = \begin{pmatrix} 1 & 2 & -2 \\ 3 & 3 & 3 \\ 1 & 1 & 0 \end{pmatrix} \qquad |A+B| = 0 + 6 - 6 - (-6 + 3 + 0) = 3 \neq 0$$

$$Adj(A+B) = \begin{pmatrix} \begin{vmatrix} 3 & 3 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 3 & 3 \\ 1 & 0 \end{vmatrix} & \begin{vmatrix} 3 & 3 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & -2 \\ 1 & 0 \end{vmatrix} & \begin{vmatrix} 1 & -2 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & -2 \\ 3 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & -2 \\ 3 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} -3 & 3 & 0 \\ -2 & 2 & 1 \\ 12 & -9 & -3 \end{pmatrix} \rightarrow (Adj(A+B))^t = \begin{pmatrix} -3 & -2 & 12 \\ 3 & 2 & -9 \\ 0 & 1 & -3 \end{pmatrix}$$

$$(A+B)^{-1} = \frac{1}{3}\begin{pmatrix} -3 & -2 & 12 \\ 3 & 2 & -9 \\ 0 & 1 & -3 \end{pmatrix}$$

$$\Rightarrow X = 3(A+B)^{-1} = 3 \cdot \frac{1}{3}\begin{pmatrix} -3 & -2 & 12 \\ 3 & 2 & -9 \\ 0 & 1 & -3 \end{pmatrix} = \begin{pmatrix} -3 & -2 & 12 \\ 3 & 2 & -9 \\ 0 & 1 & -3 \end{pmatrix}$$