

MATEMÁTICAS CCSS II
ÁLGEBRA
PROBLEMA 8

SEPTIEMBRE 2011 B

Problema 1. Sean las matrices $A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix}$ y $D = \begin{pmatrix} 8 & 8 \\ 8 & 3 \end{pmatrix}$.

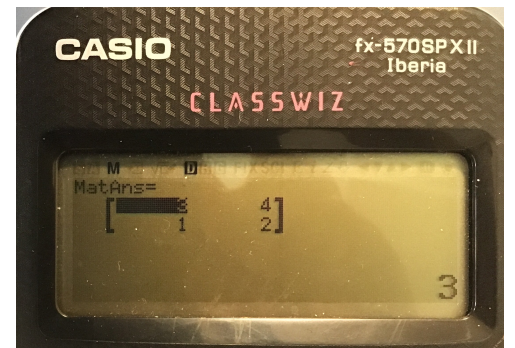
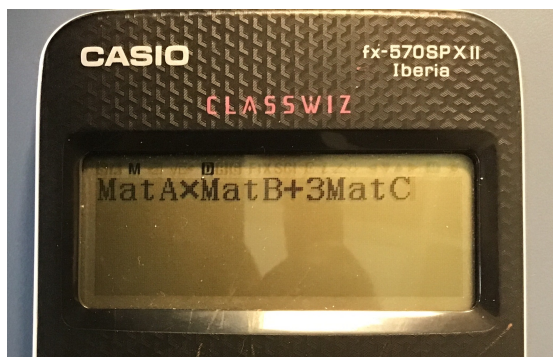
a) Calcula $AB+3C$.

b) Determina la matriz X que verifica que $AX+I=D$, donde I es la matriz identidad.

a)

$$AB+3C = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix} + 3 \begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} -3+0 & 6+1 \\ -2+0 & 4+4 \end{pmatrix} + \begin{pmatrix} 6 & -3 \\ 3 & -6 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 7 \\ -2 & 8 \end{pmatrix} + \begin{pmatrix} 6 & -3 \\ 3 & -6 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$$



b) $AX+I=D \rightarrow AX=D-I \rightarrow A^{-1}AX=A^{-1}(D-I) \rightarrow X=A^{-1}(D-I)$

Cálculo de A^{-1} : $|A| = \begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} = 12-2 = 10 \neq 0$

$\text{Adj}(A) = \begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix} \rightarrow [\text{Adj}(A)]^t = \begin{pmatrix} 4 & -1 \\ -2 & 3 \end{pmatrix} \rightarrow A^{-1} = \frac{1}{10} \begin{pmatrix} 4 & -1 \\ -2 & 3 \end{pmatrix}$

$$\Rightarrow X = \frac{1}{10} \begin{pmatrix} 4 & -1 \\ -2 & 3 \end{pmatrix} \left[\begin{pmatrix} 8 & 8 \\ 8 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] =$$

$$= \frac{1}{10} \begin{pmatrix} 4 & -1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 7 & 8 \\ 8 & 2 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 28-8 & 32-2 \\ -14+24 & -16+6 \end{pmatrix}$$

$$= \frac{1}{10} \begin{pmatrix} 20 & 30 \\ 10 & -10 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$$

