

MATEMÁTICAS CCSS II
ÁLGEBRA
PROBLEMA 10

JUNIO 2012 B

Problema 1. Dadas matrices $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ y $B = \begin{pmatrix} 2 & -6 \\ -1 & -2 \end{pmatrix}$, obtén todas las matrices de la forma $X = \begin{pmatrix} x & 0 \\ y & z \end{pmatrix}$ que satisfacen la relación $AX - XA = B$.

$$\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} x & 0 \\ y & z \end{pmatrix} - \begin{pmatrix} x & 0 \\ y & z \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -6 \\ -1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} x+2y & 2z \\ -x+3y & 3z \end{pmatrix} - \begin{pmatrix} x & 2x \\ y-z & 2y+3z \end{pmatrix} = \begin{pmatrix} 2 & -6 \\ -1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 2y & 2z-2x \\ -x+2y+z & -2y \end{pmatrix} = \begin{pmatrix} 2 & -6 \\ -1 & -2 \end{pmatrix}$$

$$\left. \begin{array}{l} 2y = 2 \rightarrow y = 1 \\ 2z - 2x = -6 \\ -x + 2y + z = -1 \\ -2y = -2 \rightarrow y = 1 \end{array} \right\} \rightarrow \left. \begin{array}{l} 2z - 2x = -6 \xrightarrow{\div 2} -x + z = -3 \\ -x + 2 + z = -1 \\ \underline{-x + z = -3} \end{array} \right\} \begin{array}{l} \text{Llamo } x = \lambda, \lambda \in \mathbb{R} \\ z = -3 + \lambda \end{array}$$

$$X = \begin{pmatrix} x & 0 \\ y & z \end{pmatrix} = \begin{pmatrix} \lambda & 0 \\ 1 & -3 + \lambda \end{pmatrix}$$

$$\lambda \in \mathbb{R}$$