

MATEMÁTICAS CCSS II
ÁLGEBRA
PROBLEMA 19

JULIO 2014 A

Problema 1. Dos matrices A y B satisfacen las siguientes igualdades:

$$A+B = \begin{pmatrix} 5 & 3 \\ 3 & 0 \end{pmatrix}, \quad A-B = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$$

- a) Calcula A y B .
 b) Calcula la matriz X sabiendo que $AXA = B$.

a)

$$A+B = \begin{pmatrix} 5 & 3 \\ 3 & 0 \end{pmatrix}$$

$$A-B = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$$

sumamos

$$2A = \begin{pmatrix} 6 & 4 \\ 2 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix} + B = \begin{pmatrix} 5 & 3 \\ 3 & 0 \end{pmatrix} \rightarrow B = \begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix}$$

b)

$$AXA = B$$

~~$$A^{-1}AXA = A^{-1}B$$~~

~~$$XAA^{-1} = A^{-1} \cdot B \cdot A^{-1}$$~~

$$X = A^{-1} \cdot B \cdot A^{-1}$$

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix} \quad |A| = -2$$

$$\left(\begin{array}{l} \text{Adj}(A) = \begin{pmatrix} 0 & -1 \\ -2 & 3 \end{pmatrix} \quad (\text{Adj}(A))^T = \begin{pmatrix} 0 & -2 \\ -1 & 3 \end{pmatrix} \\ A^{-1} = \frac{1}{-2} \begin{pmatrix} 0 & -2 \\ -1 & 3 \end{pmatrix} \end{array} \right)$$

$$X = -\frac{1}{2} \begin{pmatrix} 0 & -2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} \cdot \left(-\frac{1}{2}\right) \begin{pmatrix} 0 & -2 \\ -1 & 3 \end{pmatrix} =$$

$$= \frac{1}{4} \begin{pmatrix} 0 & -2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & -2 \\ -1 & 3 \end{pmatrix} =$$

$$= \frac{1}{4} \begin{pmatrix} -4 & 0 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} 0 & -2 \\ -1 & 3 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 0 & 8 \\ 1 & -11 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 1/4 & -11/4 \end{pmatrix}$$