MATEMÁTICAS CCSS II
ÁLGEBRA **JUNIO 2016 A**
PROBLEMA 25

Problema 1. Sean las matrices $A=\begin{pmatrix}1&2&1\\1&3&1\\0&1&3\end{pmatrix}$ y $B=\begin{pmatrix}0&-1&2\\1&0&-1\\2&1&0\end{pmatrix}$.

a) Calcula A^{-1}.

b) Determina la matriz X tal que $AX=A+B$.

a) $|A|=\begin{vmatrix}1&2&1\\1&3&1\\0&1&3\end{vmatrix}=9+1+0-(0+1+6)=3\neq 0 \quad \exists A^{-1}$

$$Adj(A)=\begin{pmatrix}\begin{vmatrix}3&1\\1&3\end{vmatrix} & -\begin{vmatrix}1&1\\0&3\end{vmatrix} & \begin{vmatrix}1&3\\0&1\end{vmatrix}\\ -\begin{vmatrix}2&1\\1&3\end{vmatrix} & \begin{vmatrix}1&1\\0&3\end{vmatrix} & -\begin{vmatrix}1&2\\0&1\end{vmatrix}\\ \begin{vmatrix}2&1\\3&1\end{vmatrix} & -\begin{vmatrix}1&1\\1&1\end{vmatrix} & \begin{vmatrix}1&2\\1&3\end{vmatrix}\end{pmatrix}=\begin{pmatrix}8&-3&1\\-5&3&-1\\-1&0&1\end{pmatrix}$$

$$(Adj(A))^t=\begin{pmatrix}8&-5&-1\\-3&3&0\\1&-1&1\end{pmatrix} \rightarrow \quad A^{-1}=\frac{1}{3}\begin{pmatrix}8&-5&-1\\-3&3&0\\1&-1&1\end{pmatrix}=\begin{pmatrix}8/3&-5/3&-1/3\\-1&1&0\\1/3&-1/3&1/3\end{pmatrix}$$

b) $AX=A+B \rightarrow A^{-1}AX=A^{-1}(A+B)$

$\rightarrow X=A^{-1}(A+B)$

$$X=\frac{1}{3}\begin{pmatrix}8&-5&-1\\-3&3&0\\1&-1&1\end{pmatrix}\left[\begin{pmatrix}1&2&1\\1&3&1\\0&1&3\end{pmatrix}+\begin{pmatrix}0&-1&2\\1&0&1\\2&1&0\end{pmatrix}\right]=$$

$$=\frac{1}{3}\begin{pmatrix}8&-5&-1\\-3&3&0\\1&-1&1\end{pmatrix}\begin{pmatrix}1&1&3\\2&3&2\\2&2&3\end{pmatrix}=\frac{1}{3}\begin{pmatrix}8-10-2&8-15-2&24-10-3\\-3+6&-3+9&-9+6\\1-2+2&1-3+2&3-2+3\end{pmatrix}=$$

$$=\frac{1}{3}\begin{pmatrix}-4&-9&11\\3&6&-3\\1&0&4\end{pmatrix}=\begin{pmatrix}-4/3&-3&11/3\\1&2&-1\\1/3&0&4/3\end{pmatrix}$$