

MATEMÁTICAS CCSS II
 ÁLGEBRA
 PROBLEMA 34

JUNIO 2018 B

Problema 1. Dadas las matrices $A = \begin{pmatrix} 2 & -1 & 5 \\ 3 & 1 & -2 \\ 5 & 1 & 3 \end{pmatrix}$ y $C = \begin{pmatrix} 7 & 4 & 1 \\ 1 & -1 & 4 \\ 8 & 4 & 6 \end{pmatrix}$, se pide:

a) Calcula A^{-1} .

(5 puntos)

b) Calcula una matriz X , de orden 3×3 , que cumpla $AX = C$.

(5 puntos)

$$a) |A| = \begin{vmatrix} 2 & -1 & 5 \\ 3 & 1 & -2 \\ 5 & 1 & 3 \end{vmatrix} = 6 + 15 + 10 - (25 - 4 - 9) = 6 + 13 = 19 \neq 0 \quad \exists A^{-1}$$

$$Adj(A) = \begin{pmatrix} \begin{vmatrix} 1 & -2 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 3 & -2 \\ 5 & 3 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 5 & 1 \end{vmatrix} \\ -\begin{vmatrix} -1 & 5 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 2 & 5 \\ 5 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 5 & 1 \end{vmatrix} \\ \begin{vmatrix} -1 & 5 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} 2 & 5 \\ 3 & -2 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 5 & -19 & -2 \\ 8 & -19 & -7 \\ -3 & 19 & 5 \end{pmatrix}$$

$$(Adj(A))^t = \begin{pmatrix} 5 & 8 & -3 \\ -19 & -19 & 19 \\ -2 & -7 & 5 \end{pmatrix} \rightarrow A^{-1} = \frac{1}{19} \begin{pmatrix} 5 & 8 & -3 \\ -19 & -19 & 19 \\ -2 & -7 & 5 \end{pmatrix} = \begin{pmatrix} 5/19 & 8/19 & -3/19 \\ -1 & -1 & 1 \\ -2/19 & -7/19 & 5/19 \end{pmatrix}$$

b) $AX = C$

$$A^{-1}AX = A^{-1}C \rightarrow X = A^{-1} \cdot C = \frac{1}{19} \begin{pmatrix} 5 & 8 & -3 \\ -19 & -19 & 19 \\ -2 & -7 & 5 \end{pmatrix} \begin{pmatrix} 7 & 4 & 1 \\ 1 & -1 & 4 \\ 8 & 4 & 6 \end{pmatrix} =$$

$$= \frac{1}{19} \begin{pmatrix} 35 + 8 - 24 & 20 - 8 - 12 & 5 + 32 - 18 \\ -133 - 19 + 152 & -76 + 19 - 76 & -19 - 76 + 114 \\ -14 - 7 + 40 & -8 + 7 + 20 & -2 - 28 + 30 \end{pmatrix} =$$

$$= \frac{1}{19} \begin{pmatrix} 19 & 0 & 19 \\ 0 & -133 & 19 \\ 19 & 19 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -7 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$AX = C$

$X \cdot A = C$

~~$A^{-1}AX = A^{-1}C$~~
 $X = A^{-1}C$

$X \cdot AA^{-1} = C \cdot A^{-1}$
 $X = C \cdot A^{-1}$